COMPARISON OF DYNAMIC ANALYSIS WITH BUILDING CODE REQUIREMENTS

by

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SYNOPSIS

The lateral seismic load requirements of the National Building Code of Canada and the Uniform Building Code, USA, are briefly summarized. The basic equations of free undamped vibration are given in matrix form and the procedure for evaluating the design shears from the results of an elastic modal analysis are outlined. The concept of a reduced stiffness matrix, which takes into account the full stiffness properties of the structure is explained. The building code design shears are compared with those of a dynamic analysis for seven different structures, of which two have actually been built. For the dynamic analysis, the structure was analyzed both as a frame and as a shear cantilever rod in most cases. From the comparative results, conclusions are drawn about the appropriate magnitude of the ductility factor, the degree of accuracy in the assumption of infinitely rigid floors, the margin of safety in the building code requirements, etc.

1. BUILDING CODE REQUIREMENTS

The lateral earthquake loads of the example structures given in this paper were evaluated in accordance with both the National Building Code of Canada[1] (NBC), 1965, and the Recommended Lateral Force Requirements of the Structural Engineers Association of California (SEAOC), 1959. The provisions of the latter recommendations are also incorporated in the Uniform Building Code[2] (UBC), USA. For the sake of completeness, the basic requirements of these two building codes, as well as the assumptions made in their application are briefly summarized below:

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1 Anonymous: "The National Building Code of Canada, 1965," Ottawa, NRC No. 8305.

2 Blume, J.A., Newmark, N.M. and Corning, L.H.: "Design of Multistory Reinforced Concrete Buildings for Earthquake Motions," Portland Cement Association, Chicago, 1961.

1.1. The National Building Code of Canada, 1965

The design base shear, V, is calculated from

$$V = R^{\circ}C^{\circ}I^{\circ}F \quad \frac{.25}{9+N} \quad W \qquad (1)$$

in which R is the zone factor and is equal to 0, 1, 2 and 4 for earthquake zones[3] 0, 1, 2 and 3, respectively; C is the construction factor equal to 0.75 or 1.25; I is the important factor equal to 1 or 1.3; F is the foundation factor equal to 1.0 or 1.5; N is the number of storeys and W is the total dead weight of the structure.

The total lateral force, V, is distributed over the height of the building in accordance with the expression

$$F_X = \frac{V}{\Sigma wh} \quad w_X h_X \qquad (2)$$

in which F_x is the lateral force acting at level x, w_x is the portion of W located at level x and h_x is the height in feet above the base. At each level the force F_x is applied over the area of the building in accordance with the mass distribution at that level.

1.2. The Uniform Building Code, USA

The total base shear is given by

$$V = K \frac{.05}{\sqrt[3]{T}} \qquad (3)$$

where K is the construction factor, which varies between 0.67 and 1.50 and represents the ability of the structure to behave in the plastic range, and T is the fundamental period of vibration of the structure in seconds in the direction under consideration. In the absence of an appropriate dynamic analysis, T may be determined from

$$T = .05H/\sqrt{D} \qquad (4)$$

in which H is the height of the main portion of the building in feet above the base and D is the dimension of the building in feet in a direction parallel to the applied forces. For one and two storey buildings, the base shear, V, is equal to 0.1 KW.

The distribution of the total base shear over the height of the building is determined in accordance with Eq.2.

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[3] Anonymous: "Supplements to National Building Code of Canada 1965", Ottawa, NRC, No. 8305X.

2. DYNAMIC ANALYSIS

As there is sufficient literature available on the dynamics of structures[4,5] and elastic modal analysis[2], the following discussion is restricted to the basic expressions of the vibration of multidegree elastic systems and to the steps involved in obtaining the design shears by means of modal superposition. For convenient computer application, matrix notation is used.

In obtaining the fundamental periods and normal mode shapes, the structure is assumed to be undamped and to behave elastically. The effects of damping and inelastic behaviour, however, are subsequently taken into account when selecting the response spectrum curves as well as in establishing an appropriate ductility factor for the structure.

2.1. Natural Periods and Mode Shapes

The equations of motion for an undamped elastic multidegree lumped-mass system subject to inertia forces acting at the joints may be expressed as

$$[M] \ \{D\} = - [K] \ \{D\} \qquad (5)$$

in which M is the diagonal mass matrix, K is the reduced stiffness matrix of the system (Eq.14) corresponding only to the vibrating directions and D is the column vector of joint displacements along the directions of vibration. Assuming an exponential trial solution of the form

$$D = \text{Real } (Xe^{i\omega t}) \qquad (6)$$

Eq.5 yields the frequency equation

$$\left[[K] - \omega^2 [M]\right] \ \{X\} = 0 \qquad (7)$$

The square root of the eigenvalues of the coefficient determinant of Eq.7 are the natural circular frequencies of the system, and the corresponding eigenvectors constitute the normal mode shapes. For convenient solution the unsymmetrical coefficient determinant of the frequency equation is converted into the following symmetrical form[6]:

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[4] Pestel, E.C., and Leckie, F.A: "Matrix Methods in Elastomechanics," McGraw-Hill Book Co., Inc., 1963.

[5] Biggs, J.M: "Introduction to Structural Dynamics," McGraw-Hill Book Co. Inc., 1964.

[6] Tezcan, S.S.: "Earthquake Analysis of Space Structures by Digital Computers," Proceedings of the Third World Conference on Earthquake Engineering, Wellington, New Zealand, p.II/T/16, January, 1965.

$$\left|[H] - \omega^2 \; [I]\right| \; \{X'\} = 0 \tag{8}$$

in which I is the identity matrix and H is a symmetrical matrix given by

$$[H] = [M]^{-1/2} \; [K] \; [M]^{-1/2}$$

$$\text{and} \qquad \{X'\} = [M]^{1/2} \; \{X\} \tag{9}$$

Once the natural circular frequencies w_i are obtained for each mode, the natural periods, T_i, are calculated from

$$T_i = 2\pi/\omega_i \tag{10}$$

2.2. Reduced Stiffness Matrix

The stiffness matrix of a framed structure such as that shown in Fig. 1 may be written in terms of the joing deformations X and joint loads F as follows:

$$\overset{A \qquad\qquad\qquad B}{\underset{B^T \qquad\qquad\qquad D}{\left[\begin{array}{cc|cc} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ \hline K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{array}\right]}} \left\{\begin{array}{c} X_1 \\ X_2 \\ \hline X_3 \\ X_4 \end{array}\right\} \begin{array}{l} X \\ \\ X_r \end{array} = \left\{\begin{array}{c} F_1 \\ F_2 \\ \hline F_3 \\ F_4 \end{array}\right\} \begin{array}{l} F \\ \\ 0 \end{array} \tag{11}$$

Assuming, as is commonly done, that the masses lumped at the corners do possess any rotational moments of inertia so that the inertia forces F_3 and F_4 are zero, and using the notations shown beside the partitioned matrices, the above equation may be written in the form

$$AX + BX_r = F \tag{12a}$$

$$CX + DX_r = 0 \tag{12b}$$

in which X_r is the column vector of joing deformations along which no vibrations take place. Solving Eq. 12b for X_r, and substituting its value into Eq. 12a, the stiffness equation (Eq. 11) is simplified to

$$[K] \; \{X\} = \{F\} \tag{13}$$

in which K is the reduced stiffness matrix given by

$$K = A - BC^{-1} B^{T} \qquad (14)$$

In applying the above formulation the joint deformations must be carefully numbered so that the main stiffness matrix is generated in the form required by Eq.11. The generation may be conveniently performed by means of the code number technique[7].

The reduced stiffness matrix concept is very useful, because only by this procedure is it possible to take into account the full stiffness properties of the structural members. By doing so, the accuracy of both the natural periods and the mode shapes is greatly improved.

2.3 Modal Superposition

At first, the eigenvalues and corresponding eigenvectors are arranged in the descending order of natural periods, after which the modal forces acting on the masses are obtained for each mode of vibration from the following relation:

$$\{F\}_i = [K] \{X\}_i \alpha_i S_{di} \qquad (15)$$

in which

F_i = the column vector of inertia forces acting on a mass along the direction of vibration in the ith mode,

K = the reduced stiffness matrix of the system obtained from Eq.14,

X_i = the column vector of displacements in the ith mode obtained from the solution of Eq. 7 (ith eigenvector),

α_i = Participation factor for the ith mode,

S_{di} = Displacement spectrum for the ith mode.

The participation factor, α_i, is calculated for each mode from[2]

$$\alpha_i = \sum_j m_j x_{ij} / \sum m_j x_{ij} \qquad (16)$$

in which m_j is the jth mass and x_{ij} is the modal displacement of the jth mass in the ith mode.

[7]Tezcan, S.S.: Discussion of "Simplified Formulation of Stiffness Matrices" by P.M. Wright, Jour. of Structural Division, ASC E, Vol. 89, No. ST6, Proc. Paper 3743, December, 1963, pp. 445-449.

The response spectrum used in the analysis of the example structures was the idealized response spectrum of the N-S component of the 1940 El Centro earthquake[2]. The idealized response spectrum curves for three groups of critical damping percentages are shown in Figure 2.

2.4. Design Shears

From the inertia forces acting on each mass, the storey shears are evaluated for each mode. Once the storey shears are determined at each level for each mode, the square root of the sum of the squares of the modal components yields the maximum probable shears. It should be remembered that these maximum probable shears correspond to an analysis based on the assumption that the structure is perfectly elastic. However, the observed behaviour of buildings during strong motion earthquakes indicates that most structures can undergo some plastic deformation without excessive damage. This is because of the fact that most structures, whether of steel or reinforced concrete, possess considerable ductility. Although it is not generally permissible to allow plastic deformations under normal conditions, it is, in most cases, acceptable to let the structure go into the plastic range under the dynamic loads of a severe earthquake which will occur only once or at most a few times during its life. Therefore, the maximum probable shears resulting from the elastic analysis are divided by a reduction factor, μ, called the "ductility factor" to yield the shears corresponding to an elasto-plastic response. The ductility factor, μ, is defined as the ratio of the ultimate displacement to the yield displacement, and reflects the degree of plastic deformation permitted. The value of μ depends upon the function of the element, type of the structure and the amount of damage that can be tolerated. A ductility factor of the order of 4 to 5 is considered reasonable for moment resisting steel and reinforced concrete frames. The ductility factor may be taken as low as 1.0 if many applications of the highest load are expected or if no damage can be tolerated. In such a case the structure always remains in the elastic range. Similarly, if the purpose of the design is to prevent collapse under a very remote chance of severe load, the ductility factor may be taken as high as 8 or 10. The shears obtained by dividing the elastic analysis values by the ductility factor should be used to design the structure with yield stresses. However, the usual practice is to work with allowable stresses augmented by a factor of 1.33 rather than with the yield stresses. Therefore, in addition to the ductility factor, a further reduction factor equal to the ratio of the yield stress to the augmented allowable stress is applied to the maximum probable shears. In short, the design shears, V_{design}, corresponding to an elasto-plastic analysis are obtained from the elastic analysis shears, V_{elas}, as follows:

$$V_{design} = V_{elas}/(\mu \times S_{yield} / 1.33 \times S_{all}) \qquad (17)$$

in which S_{yield} is the yield stress and S_{all} is the allowable stress for the material.

3. EXAMPLE STRUCTURES

In order to compare the design shears recommended by the National Building Code of Canada and the Uniform Building Code with those of a dynamic analysis, several example structures of various types and configuration were selected. These are as follows:

Structure No. 1: 3-storey, 1-bay steel frame (Fig. 3).

Structure No. 2: 2-storey, 3-bay Pulping Group Building, Skeena Kraft Ltd., Prince Rupert, B.C.* (Fig. 4).

Structure No. 3: 9-storey, 4-bay Power Group Building, Northwood Pulp Ltd., Co., Prince George, B.C.* (Fig. 5).

Structure No. 4: 15-storey, 2-bay steel frame (Fig. 6).

Structure No. 5: Elevated Water Tank (Fig. 7).

Structure No. 6: Space Needle (Fig. 8).

Structure No. 7: Reinforced concrete smokestack (Fig. 9).

3.1. General Notes

The numerical values assumed for the various factors which occur in the building code and dynamic analysis computations are summarized for all seven structures in Table 1. The calculation of the design forces in accordance with the NBC and UBC recommendations, as well as the comparative design forces of the dynamic analyses, are given in Tables 2 to 8. These tables also include the difference percentages between the building code and dynamic analysis results, as well as the execution times required for the dynamic analyses using an IBM 7040. The comparative design shears of the example structures are also shown graphically in Figs. 10 to 16. The natural periods, spectral displacements and participation factors for the first five modes of each example structure are summarized in Table 9.

In calculating the maximum probable shears, only the first ten modes corresponding to the ten highest periods were used as this was considered sufficiently accurate.

3.2 Special Notes and Assumptions

(1) The importance and foundation factors were assumed to be

*Courtesy of H.A. Simons International Ltd., Consulting Engineer, Vancouver, B.C.

1.0 in order to maintain consistency when comparing the results of the National Building Code with those of the Uniform Building Code in which these factors are not taken into account.

(ii) All the frames were regarded to be moment resisting frames capable of carrying all the lateral loads. Accordingly, the construction factors in the NBC and UBC formulae were taken as 0.75 and 0.67 respectively.

(iii) In the dynamic analysis of the frames, the full stiffness properties of the members, including the rotational and axial deformations were taken into account.

(iv) The dynamic analyses of the framed structures were performed twice; once taking the frame as a whole with all its beams and columns, and then assuming the floor systems to be infinitely rigid, i.e., as a shear cantilever rod. The space needle and smokestack examples, however, were only analyzed as bending cantilevers; in the former, equivalent rotational springs were introduced to substitute for the stiffness of the horizontal bracing elements, as illustrated in Fig. 8. In the elevated tank example the cantilever idealization was not considered as it would be too unrealistic.

(v) The elevated tank was assumed to be completely full, so that it was not necessary to consider the surge of the water inside the tank.

(vi) The allowable stress was assumed to be $0.6S_{yield}$ for steel and $0.4S_{yield}$ for concrete. Hence, the ratio of the yield stress to the augmented allowable stress was 1.25 and 1.67 for steel and concrete, respectively.

3.3. Discussion of Results

(i) The value of the ductility factor was assumed to be 4 for the framed structures. This value appears to account reasonably well for the inelastic response when the frame is considered as a whole. However, the same ductility factor, when applied to the cantilever rod idealization, resulted in much higher design forces, which, in most cases, were unacceptable (Structure Nos. 1, 2, and 4). The reason that the ductility factor suitable for a frame is not suitable for the corresponding cantilever rod, is due to the fact that the cantilever idealization generally does not represent the full stiffness properties of the actual frame. The degree of accuracy of the cantilever rod results depends chiefly upon the approximation involved in assuming the floor system to be infinitely rigid. Because the beam to column stiffness ratio varies grossly from building to building Building, it appears to be inadvisable to establish a unique ductility factor for use in connection with cantilever rod idealization. To treat frames as a whole and apply a ductility factor of about 4 seems to be the best approach as it produces consistently reasonable results.

(ii) For the elevated tank, space needle and smokestack examples, a ductility factor of about 2 was used to account for very low critical damping percentages, as well as to reduce the amount of tolerable plastic deformations in these slender and important structures. In calculating the building code shears for these three structures, the number of storeys, N, was taken as 1.

(iii) The Power Group Building, shown in Fig. 5, houses a heavy recovery boiler, which is suspended from the eighth floor. The swinging of the boiler during a horizontal earthquake is restricted by earthquake stops at the floor levels. Even though the boiler is not rigidly connected to the moment resisting frames, the lateral inertia forces of the boiler are expected to be transferred to the adjoining frames at these stops, as the clearance between the boiler and the frames is only .25 to .5 inches.

In evaluating the design shears, the boiler was regarded as an attachment to the building and, therefore, in accordance with both the NBC and UBC requirements, a horizontal force of 0.1g was considered to be transferred from the boiler to the frames. At the time of the design of this building (Nov., 1964) the 1960 edition of the National Building Code was in effect, and the lateral loads calculated according to it were 35-40% higher than those recommended by the Uniform Building Code. The dynamic analysis gave design shears* far lower than even those of the UBC (Table 4), thus proving that the 1960 NBC requirements were excessively high.

(iv) A typical feature that can be observed in the design shear diagrams (Figs. 10-16) is that, in most cases, the distribution of the dynamic analysis shears throughout the height of the building follows a somewhat different pattern from that of the building codes. For the purpose of comparison, if the base shears of the dynamic analyses are made equal to those of the NBC or UBC, it will be seen that the dynamic analysis shears in the lower storeys are less than those of the building codes, while in the upper storeys, the opposite trend is noticed. The relatively higher design shears obtained for the upper storeys by a dynamic analusis are more realistic than the code values, because the local accelerations, as well as the moments and shears at higher elevations of the structure are greater than those at lower elevations.

(v) The relative magnitudes of the design shears at the top of slender structures, such as Structure Nos. 4, 5, 6 and 7, show clearly that whip action is better reflected in the dynamic analysis results than in the building code requirements.

(vi) With the exception of Structure Nos. 2 and 3, the elastic properties of the example structures and, hence, their dynamic analysis

*To take into account the fact that Prince George is in earthquake zone 2 which has a seismicity factor equal to half of that for Zone 3, the dynamic analysis results were divided by two.

results correspond to a first trial. Doubtless, further trials would be necessary to arrive at economical designs.

4. CONCLUSIONS

1. The selection of an appropriate ductility factor for a structure is vitally important for the significance of a modal analysis. An improper choice of the ductility factor may completely offset the accuracy attained by a meticulous calculation of the maximum probable shears. Therefore, further experimental and theoretical research is necessary in order to determine suitable values for the ductility factors of various types of structures.

2. The idealization of a structure into a shear cantilever rod may lead to erroneous values for the natural periods, mode shapes and seismic forces. These errors increase in magnitude as the beam to column stiffness ratio decreases. It appears that when the frame is considered as a whole, more realistic results are obtained.

3. The comparative dynamic analuses show that the design forces corresponding to the cantilever rod idealization, which assumes infinitely rigid floors, are consistently higher than those obtained considering the frame as a whole. This illustrates the important fact that the seismic forces increase considerably with increases in the stiffness of the structure, even though the mass and all the other parameters remain constant. Therefore, contrary to what is normally expected, it may well happen that heavier sections do not necessarily increase the safety of the structure against earthquakes, because the inertia forces attracted are relatively higher. An optimum earthquake proof design can only be achieved by a trial and error procedure combined with a sound knowledge of the dynamics of structures.

4. Although the comparative design forces of the example structures show that the building code requirements are generally higher than those of the dynamic analyses and, therefore, can be considered on the safe side, their economy may sometimes be questionable. Further, it is not possible to consider varying degrees of damping or the effects of changes in the stiffness properties of the structure in the code formulae. On the other hand, an appropriate dynamic analysis can allow for these varying properties and, at the same time, provide safe and economical design criteria.

5. A particular response spectrum is not applicable to all subsoil conditions, as it depends on the properties of the underlying

soil. Therefore, for reliable dynamic analyses, sufficient data data sbout the response spectrum curves for different subsoil conditions would be desirable.

ACKNOWLEDGEMENTS

The writer wishes to express his gratitude to research assistants Ravindar K. Kinra and David A. W. Pecknold for their valuable help in preparing the paper, and to the National Research Council of Canada for the financial support.

TABLE 1. SUMMARY OF NUMERICAL FACTORS							
Struct. No.	Description	Duct. fac. μ	$\frac{S_y}{1.33\ S_{all.}}$	Critical Damping	Zone fac. R	Construction Factor	
						C NBC	K UBC
1	3-storey Frame	4	1.25	5-10%	4	.75	.67
2	Pulping Group Building	4	1.25	5-10%	4	.75	.67
3	Power Group Building Frame 52	4	1.25	2-5%	2	.75	.67
4	15-storey Frame	4	1.25	5-10%	4	.75	.67
5	Elevated Water Tank	2	1.25	0-2%	4	1.25	1.50
6	Space Needle	2	1.25	0-2%	4	1.25	1.50
7	Smokestack	1.5	1.67	0-2%	4	.75	1.00

TABLE 2. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE NO.1							
Storey No.	H ft,	W Kips	WH	Building Codes		Dynamic Analysis	
				NBC	UBC	Frame .16 min*	Cantilever .13 min*
3	36	150	5400	15.5	12.2	9.7	11.2
2	24	300	7200	20.7	16.2	7.7	15.6
1	12	300	3600	10.4	8.1	5.6	12.7
TOTAL		750	16200	46.6	36.5	23.0	39.5
DIFFERENCE				+104%	+59%	0%	+72%
PERIOD (secs)				-	.33	1.98	1.13

$V_{(NBC)} = 4 \times .75 \times \frac{.25}{9+3} \times 750 = 46.6$ k ; $V_{(UBC)} = .67 \times \frac{.05}{\sqrt[3]{.32}} \times 750 = 36.5$ k

TABLE 3. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE NO.2							
Storey No.	H ft.	W Kips	WH	Building Codes		Dynamic Analysis	
				NBC	UBC	Frame .40 min*	Cantilever .11 min*
3	63.5	160	10000	19	14	26	20
2	54.0	186	10000	19	14	20	23
1	29.75	1164	34600	65	47	52	116
TOTAL		1510	54600	103	75	98	159
DIFFERENCE				+5%	-24%	0%	+62%
PERIOD (secs)				-	.32	.82	.54

$V_{(NBC)} = 4 \times .75 \times 1 \times 1 \times \frac{.25}{9+2} \times 1510 = 103k$; $V_{(UBC)} = .67 \times \frac{.05}{\sqrt[3]{.32}} \times 1510 = 75k$

TABLE 4. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE NO.3							
Storey No.	H ft.	W Kips	WH	Building Codes		Dynamic Analysis	
				NBC +10% B.Wt.	UBC +10% B.Wt.	Frame 4.16 min*	Cantilever .37 min*
9	172.2	129	22200	5.2	4.3	3.9	3.2
8	164.2	80	13150	3.0	2.5	2.5	2.0
7	149.7	80	11970	2.8	2.2	2.3	2.0
6	130.7	78+750	10200	2.4+75.	2.0+75.	15.9	18.8
5	117.8	80+24	9420	2.2+2.4	1.8+2.4	1.6	2.2
4	102.3	91+470	9310	2.1+47.	1.8+47.	9.3	11.2
3	73.7	197+96	14520	3.3+9.6	2.9+9.6	2.5	3.4
2	37.7	214+98	8070	1.9+9.8	1.6+9.8	2.0	1.4
1	21.7	202+260	4380	1.0+26.	0.8+26.	6.0	7.1
TOTAL		$\underbrace{1151+1698}_{2849}$	103220	$\underbrace{24+169.8}_{193.8}$	$\underbrace{19.9+169.8}_{189.7}$	46.0	51.3
DIFFERENCE				+320%	+312%	0%	+11%
PERIOD (secs)				-	.90	2.81	2.44

$V_{(NBC)} = 2 \times .75 \times 1 \times 1 \times \frac{.25}{9+9} \times 1151 = 24k$; $V_{(UBC)} = .67 \times \frac{.05}{\sqrt[3]{.9}} \times 1151 = 19.9k$

TABLE 5. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE NO.4							
Storey No.	H ft.	W Kips	WH	Building Codes		Dynamic Analysis	
				NBC	UBC	Frame 9.5 min*	Cantilever .80 min*
15	150	60	9000	3.6	3.4	7.0	10.0
14	140	60	8400	3.4	3.2	4.6	8.1
13	130	64	8310	3.3	3.1	3.2	6.1
12	120	64	7700	3.2	2.9	2.3	5.8
11	110	64	7040	2.8	2.6	1.5	4.5
10	100	64	6400	2.6	2.5	1.4	4.1
9	90	64	5760	2.3	2.2	1.2	3.5
8	80	68	5440	2.2	2.0	1.2	3.7
7	70	68	4760	1.9	1.8	1.3	3.1
6	60	68	4080	1.7	1.5	1.9	3.1
5	50	68	3400	1.5	1.3	1.9	2.8
4	40	72	2880	1.2	1.1	1.5	2.7
3	30	72	2160	0.9	0.8	1.5	2.2
2	20	72	1440	0.6	0.6	1.0	1.3
1	10	72	720	0.2	0.2	0.6	1.0
T O T A L		1000	77480	31.2	29.2	32.1	62.0
D I F F E R E N C E				-2.7%	-9.0%	0%	+99%
P E R I O D (secs)				-	1.23	1.59	0.82

$V = 4 \times .75 \times 1 \times 1 \times \frac{.25}{9+15} \times 1000 = 31.2$ (NBC) ; $V = .67 \times \frac{.05}{\sqrt[3]{1.23}} \times 1000 = 29.2$ (UBC)

TABLE 6. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE No.5						
Level No.	H ft.	W Kips	WH ($x10^2$)	Building Codes		Dynamic Analysis
				NBC	UBC	Frame .34 min*
3	120	1748	2098	230	276	205
2	80	100	80	9	10.6	6
1	40	102	40	4	5.4	5
TOTAL		1950	2218	243k 12.5%W	292 15%W	216 11%W
DIFFERENCE				+12%	+35%	0%
PERIOD (secs)				-	-	2.37

$V = 4 \times 1.25 \times \frac{.25}{9+1} \times 1950 = 243k$ (NBC) ; $V = 1.5 \times .1 \times 1950 = 292k$ (UBC)

TABLE 7. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE NO.6						
Level No.	H ft.	W Kips	WH ($x10^3$)	Building Codes		Dynamic Analysis
				NBC	UBC	Cantilever .64 min*
12	600	7400	4440.0	1239	1496	978
11	550	303	166.6	47	56	8
10	500	303	151.5	43	51	-8
9	450	303	136.3	39	46	-6
8	400	686	274.0	77	94	30
7	350	661	231.0	65	77	96
6	300	711	213.0	59	72	130
5	250	661	165.2	46	56	116
4	200	661	132.2	37	44	100
3	150	736	110.3	31	37	94
2	100	661	66.1	18	22	51
1	50	661	33.0	9	11	15
TOTAL		13747	6119.2	1710 12.5%W	2062 15%W	1604 11.5%W
DIFFERENCE				+7%	+25%	0%
PERIOD (secs)				-	2.74	2.53

$V = 4 \times 1.25 \times \frac{0.25}{9+1} \times 13747 = 1710k$ (NBC) ; $V = 1.5 \times .1 \times 13747 = 2062k$ (UBC)

TABLE 8. COMPARATIVE DESIGN FORCES (KIPS), STRUCTURE NO.7						
Level No.	H ft.	W Kips	WH_2 ($x10^2$)	Building Codes		Dynamic Analysis
				NBC	UBC	Cantilever .5 min*
10	300	169.1	507.3	24	32	54
9	270	173.4	468.2	22	30	5
8	240	179.0	429.6	20	27	-6
7	210	184.6	387.7	18	25	1
6	180	188.8	339.9	16	21	4
5	150	194.3	291.4	14	19	5
4	120	200.3	240.4	12	15	11
3	90	204.3	180.3	9	11	22
2	60	210.3	126.2	6	8	27
1	30	215.7	64.7	3	4	15
T O T A L		1920	3038	144 7.5%W	192 10%W	138 7.2%W
D I F F E R E N C E				+4%	+39%	0%
P E R I O D (secs)				-	-	5.9

$V_{(NBC)} = 4 \times .75 \times \frac{.25}{9+1} \times 1920 = 144k$; $V_{(UBC)} = 1.0 \times .1 \times 1920 = 192k$

TABLE 9. PERIODS, SPECTRAL DISPLACEMENTS AND PARTICIPATION FACTORS

No.	Structure		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	Three storey frame	T	1.978	0.619	0.316	0.074	0.066
		S_d	6.190	2.200	0.600	0.035	0.025
		α	1.317	-0.405	0.147	.0005	0.001
1a	Three storey frame idealized as a cantilever	T	1.134	0.423	0.271	-	-
		S_d	3.390	1.050	0.430	-	-
		α	1.232	-0.274	-0.082	-	-
2	Pulping group building idealized as a frame	T	0.820	0.406	0.109	0.082	0.056
		S_d	2.700	0.849	0.060	0.035	0.016
		α	1.764	-0.795	0.001	-0.043	0.001
2a	Pulping group building as a cantilever	T	0.538	0.196	0.040	-	-
		S_d	1.800	0.250	0	-	-
		α	1.159	-0.159	0	-	-
3	Power group building idealized as a frame	T	2.813	0.782	0.526	0.484	0.371
		S_d	13.000	3.487	2.344	2.291	1.346
		α	1.186	0.577	0.155	-0.023	-0.055
3a	Power group building idealized as a cantilever	T	2.443	0.692	0.373	0.295	0.227
		S_d	10.886	3.085	1.357	0.849	0.505
		α	1.146	0.567	-0.067	-0.021	0.002
4	Fifteen storey frame	T	1.590	0.586	0.337	0.236	0.179
		S_d	5.200	2.000	0.800	0.400	0.220
		α	1.408	-0.606	0.338	0.254	-0.215
4a	Fifteen storey frame idealized as a cantilever	T	0.822	0.338	0.209	0.150	0.119
		S_d	2.700	0.750	0.290	0.145	0.070
		α	1.454	-0.700	0.392	0.248	-0.209
5	Elevated water tank	T	2.560	0.357	0.198	0.033	0.009
		S_d	16.784	1.645	0.504	0.003	0
		α	1.024	0.512	0.197	0	0
6	Space needle	T	2.527	0.363	0.105	0.053	0.033
		S_d	16.571	1.696	0.142	0.036	0.004
		α	1.096	0.921	-0.626	0.498	-0.303
7	Reinforced concrete smokestack	T	5.963	1.041	0.380	0.195	0.118
		S_d	16.600	6.829	1.868	0.489	0.179
		α	1.508	-0.753	0.390	0.275	-0.206

T = Period (secs) ; S_d = Spectral displ. (in) ; α = Participation factor

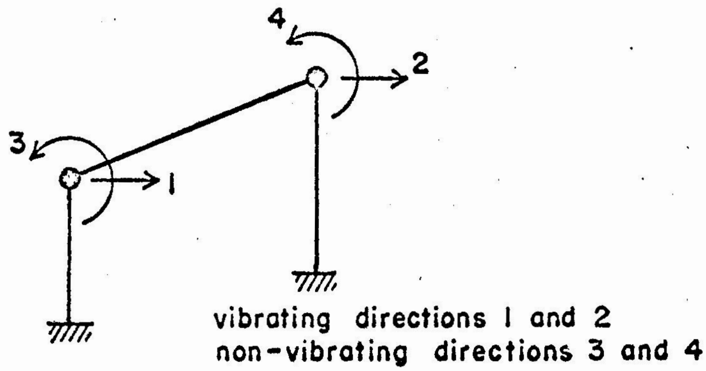


Fig.1 Deformation numbering.

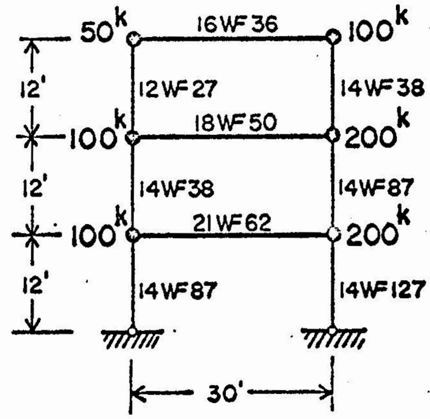


Fig.3 Three storey frame, Structure No.1.

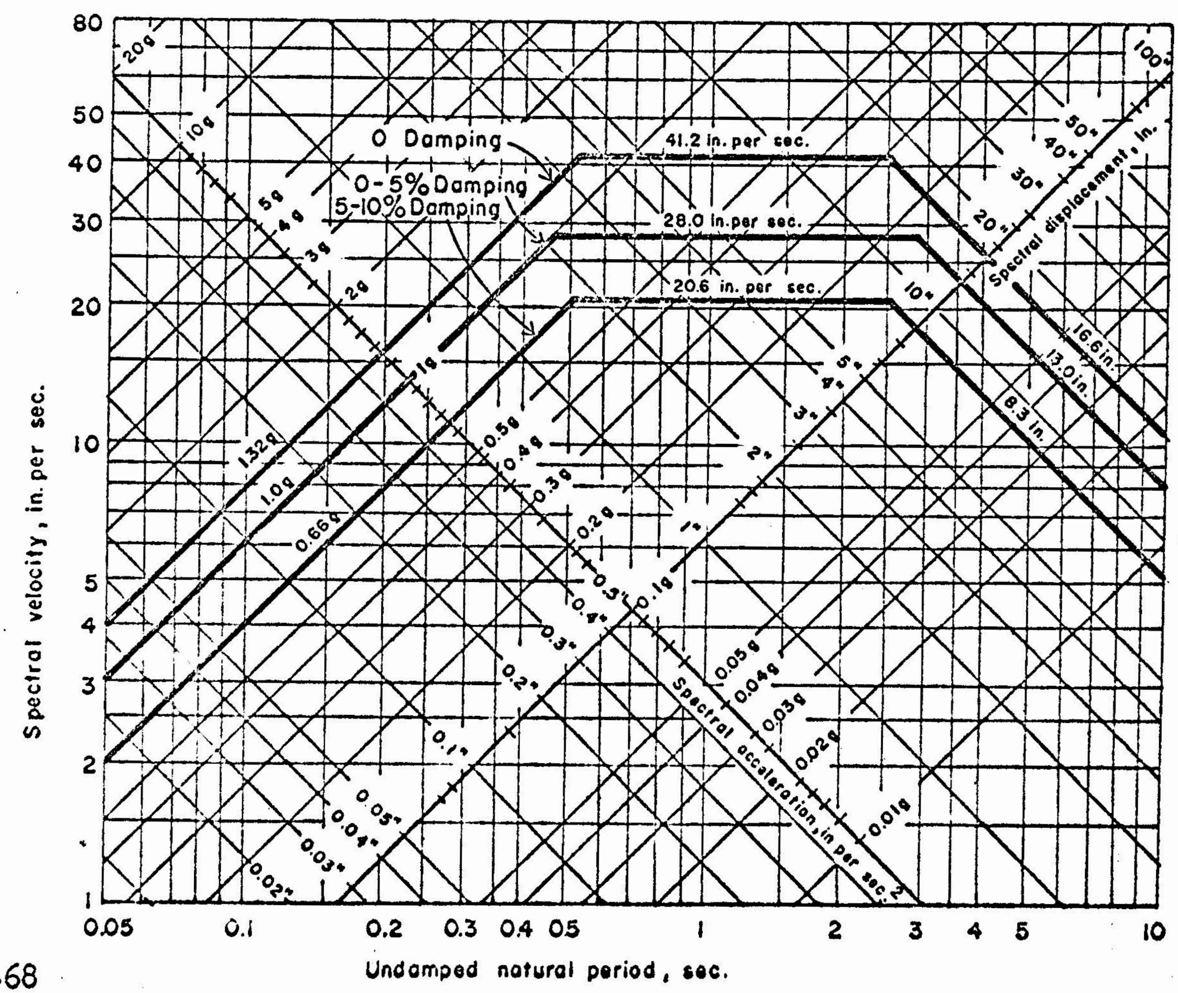


Fig.2 Idealized response spectrum, 1940 El Centro earthquake.

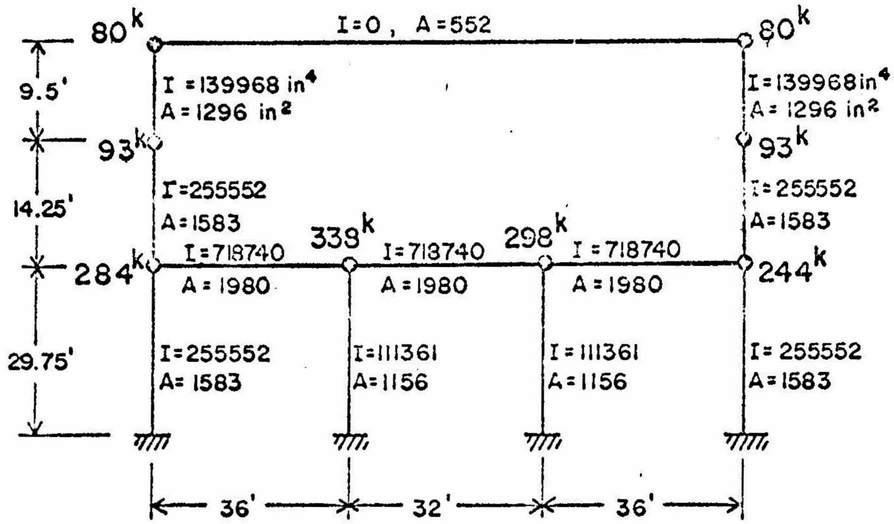
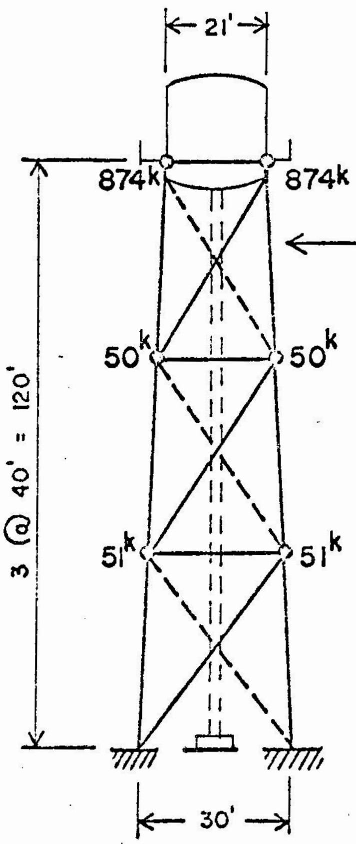


Fig.4 Pulping group building, Structure No.2.



Member	I (in⁴)	A (in²)
Columns & Ties	4610	176
Bracing	0	1
Top beam	53000	888

Fig.7 Elevated water tank, Structure No.5.

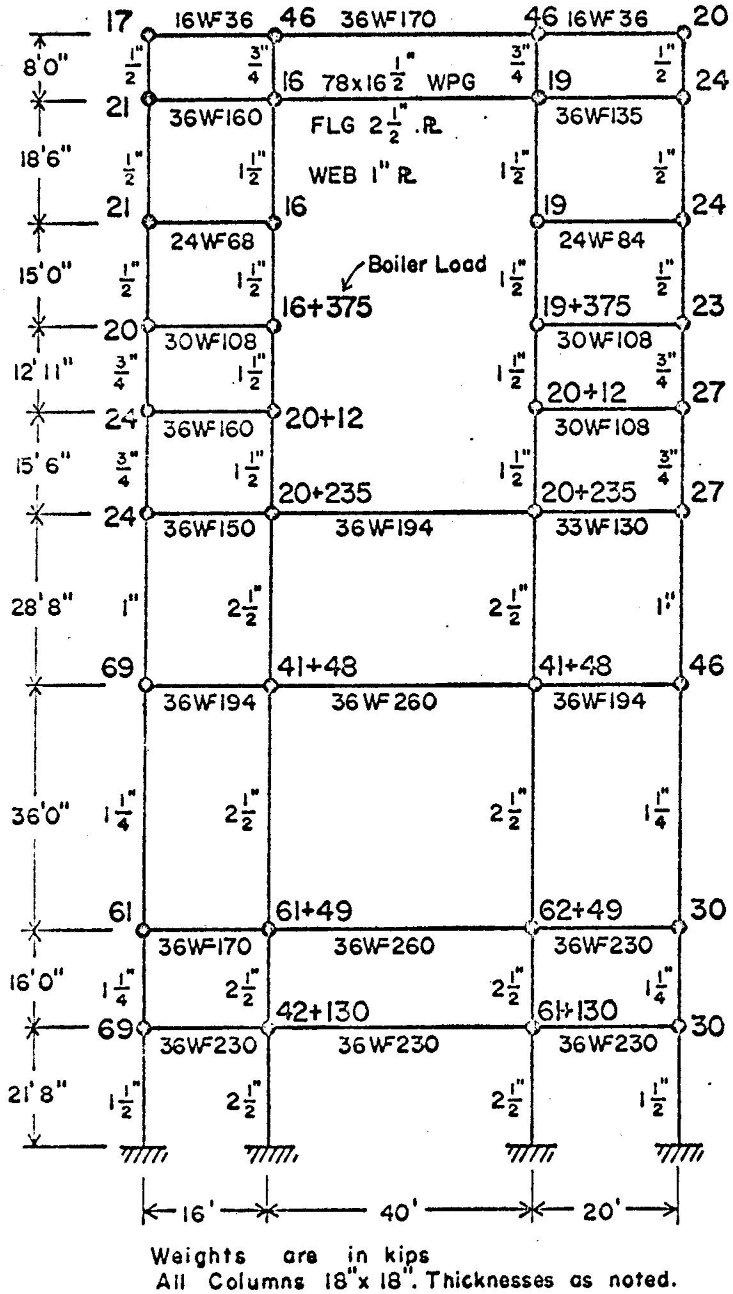


Fig.5 Power group building, Structure No.3.

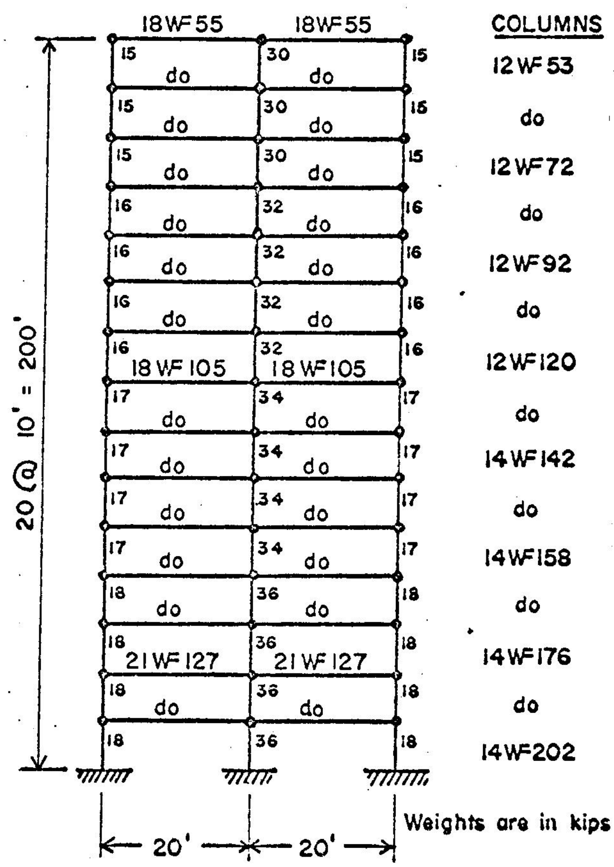
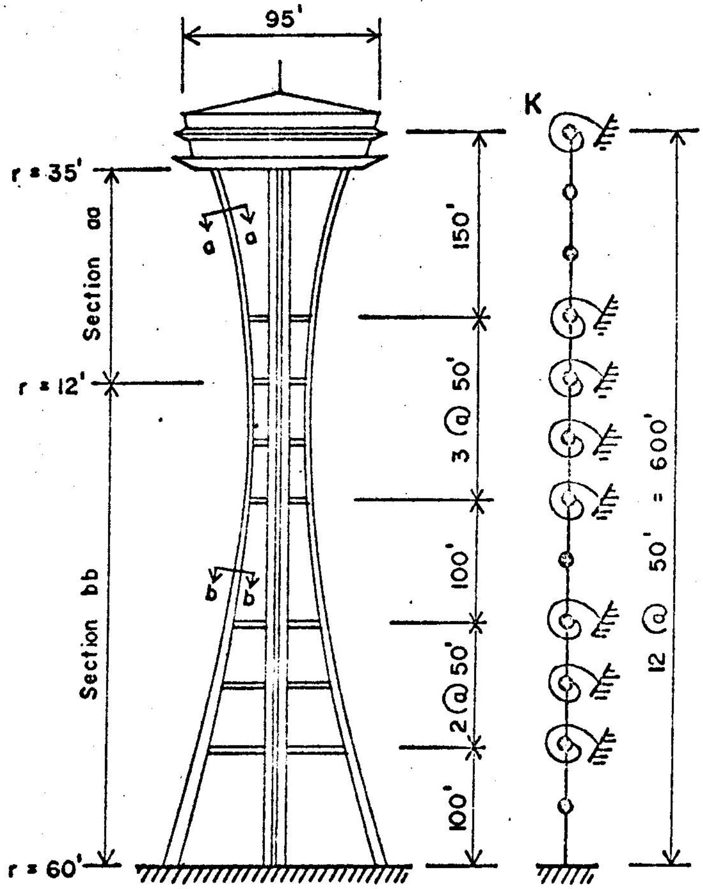


Fig.6 Fifteen storey frame, Structure No.4.



$I_x (ft^4)$	$A (ft^2)$	$K (10^4 kft)$
7,500	12.36	3000
6,500	12.36	—
5,000	12.36	—
2,500	12.36	15
5,000	27.0	15
11,000	27.0	8
17,000	27.0	7
22,500	27.0	—
28,000	27.0	5
33,500	27.0	4
40,000	27.0	3
46,000	27.0	—

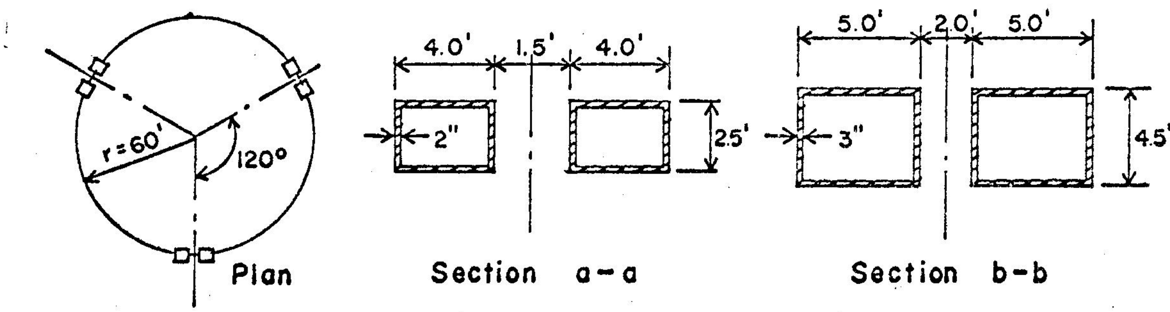
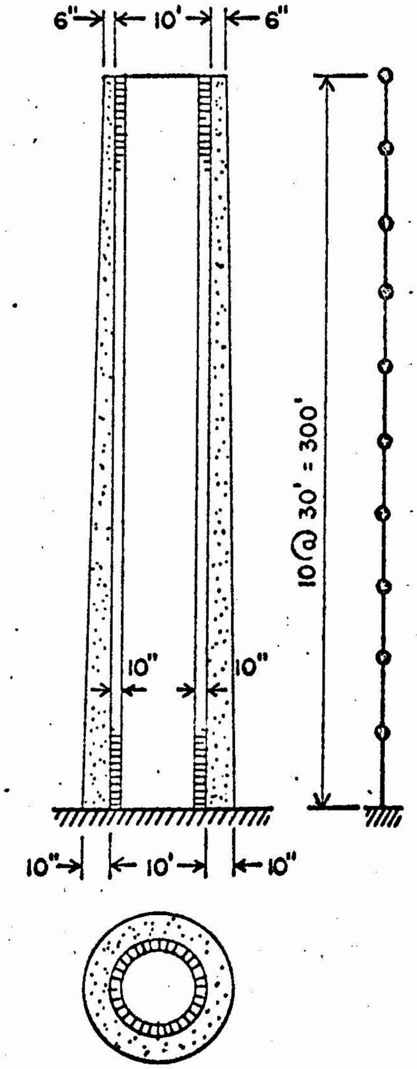


Fig.8 Space needle, Structure No.6.



$I(ft^4)$	$A(ft^2)$	W(kips)
238	17.3	169.1
254	18.2	173.4
269	19.2	179
289	20.4	184.6
308	21.7	188.8
324	22.6	194.3
349	24.2	200.3
370	25.4	204.3
385	26.4	210.3
406	27.6	215.7

Fig.9 Reinforced concrete smokestack, Structure No.7.

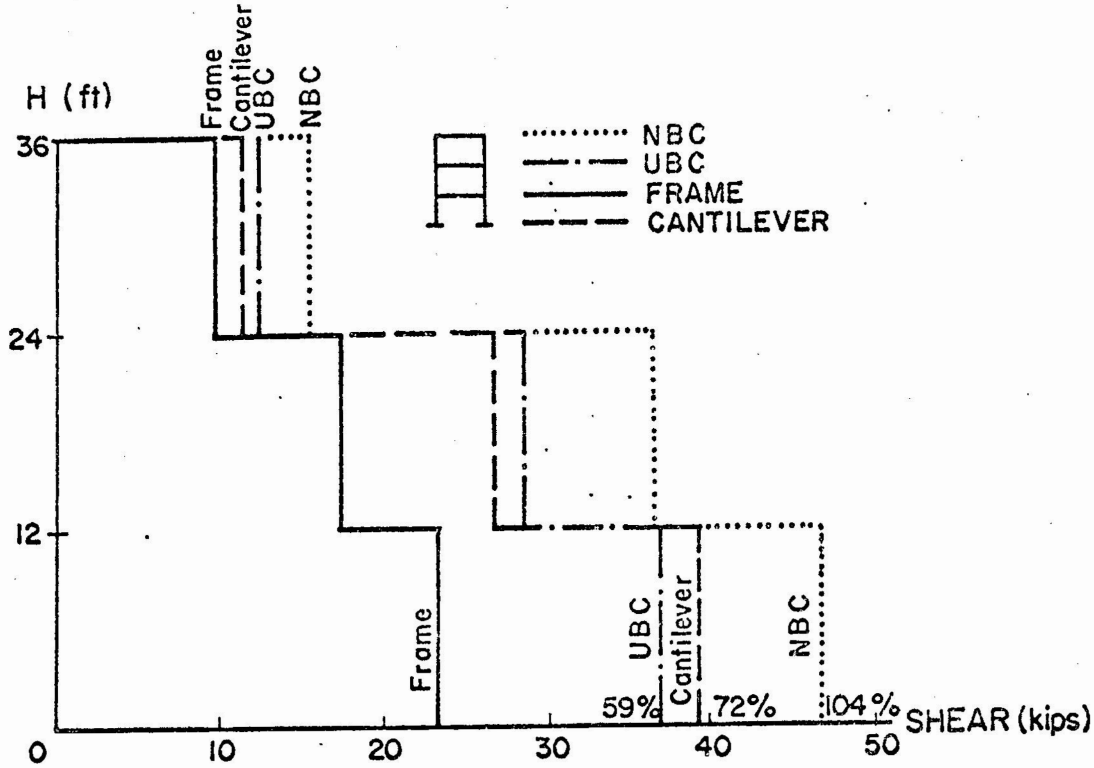


Fig.10 STRUCTURE No. 1

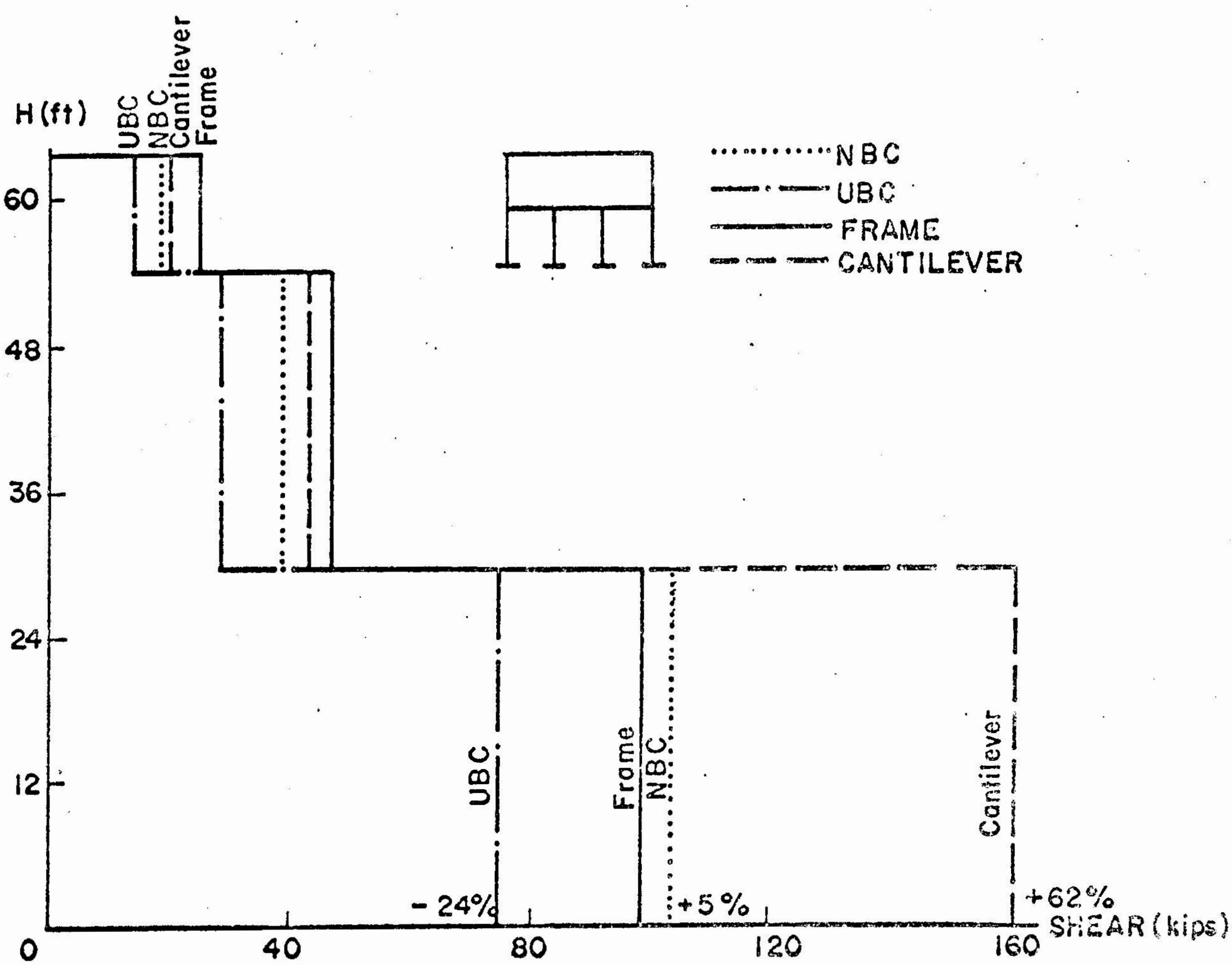


Fig.11 STRUCTURE No. 2

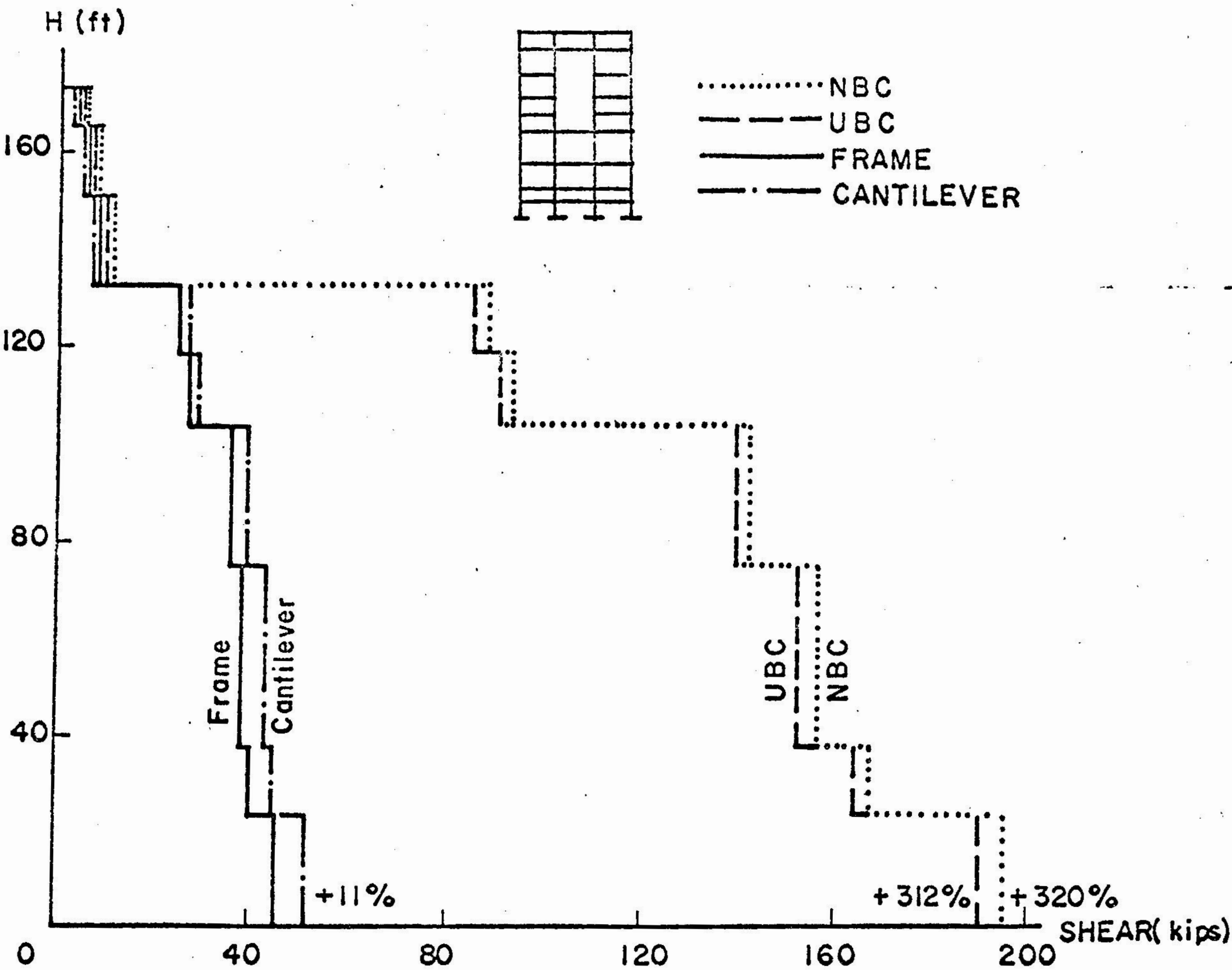


Fig.12 STRUCTURE No. 3

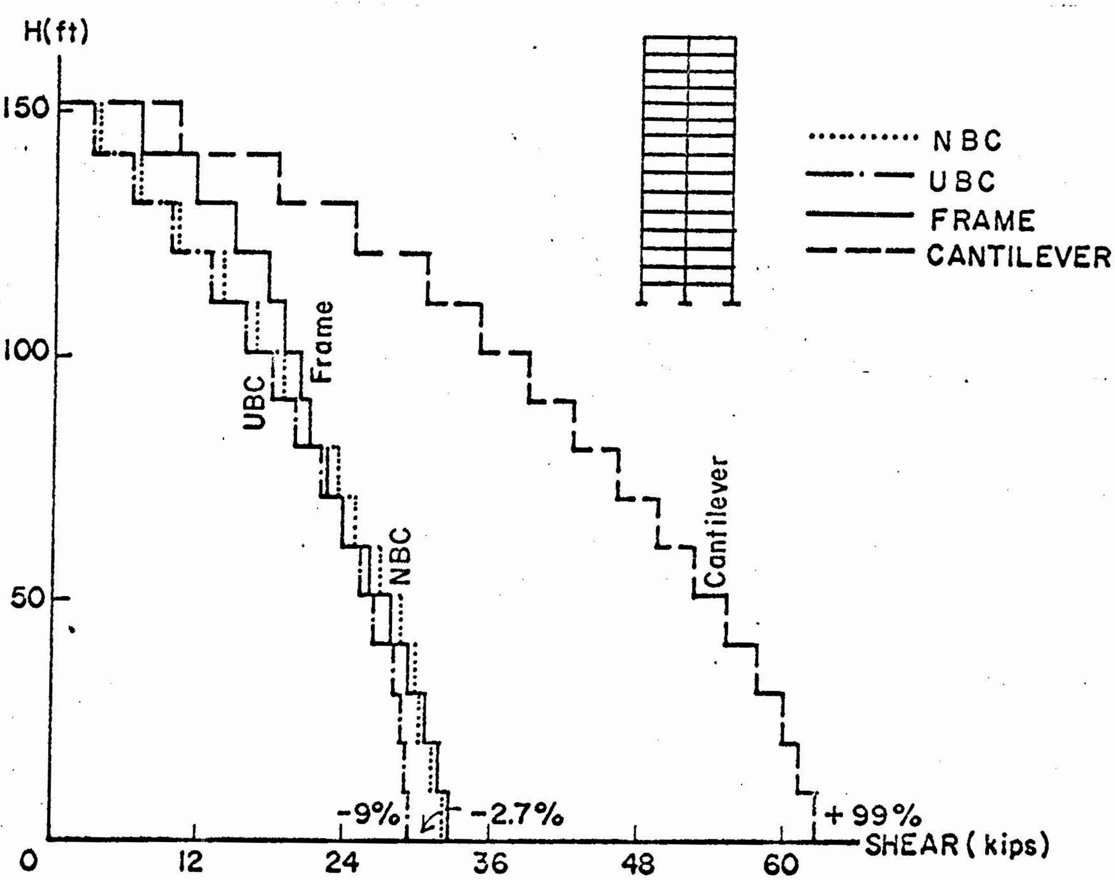


Fig.13 STRUCTURE No. 4

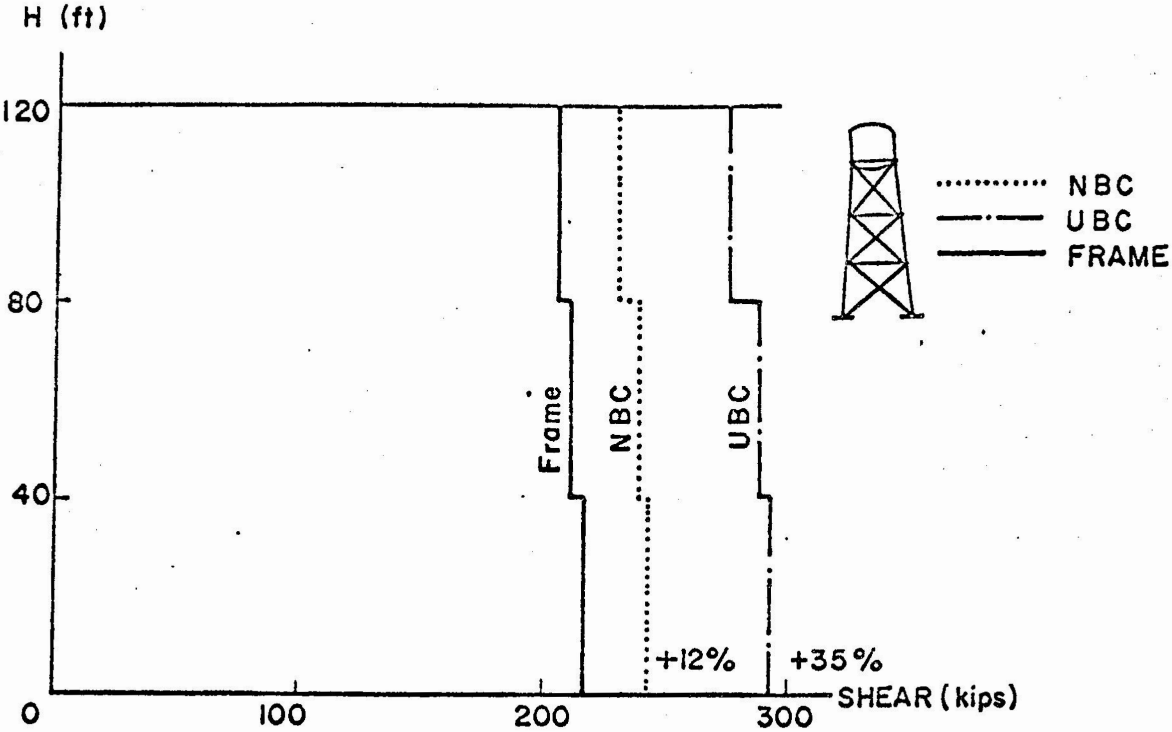


Fig.14 STRUCTURE No. 5

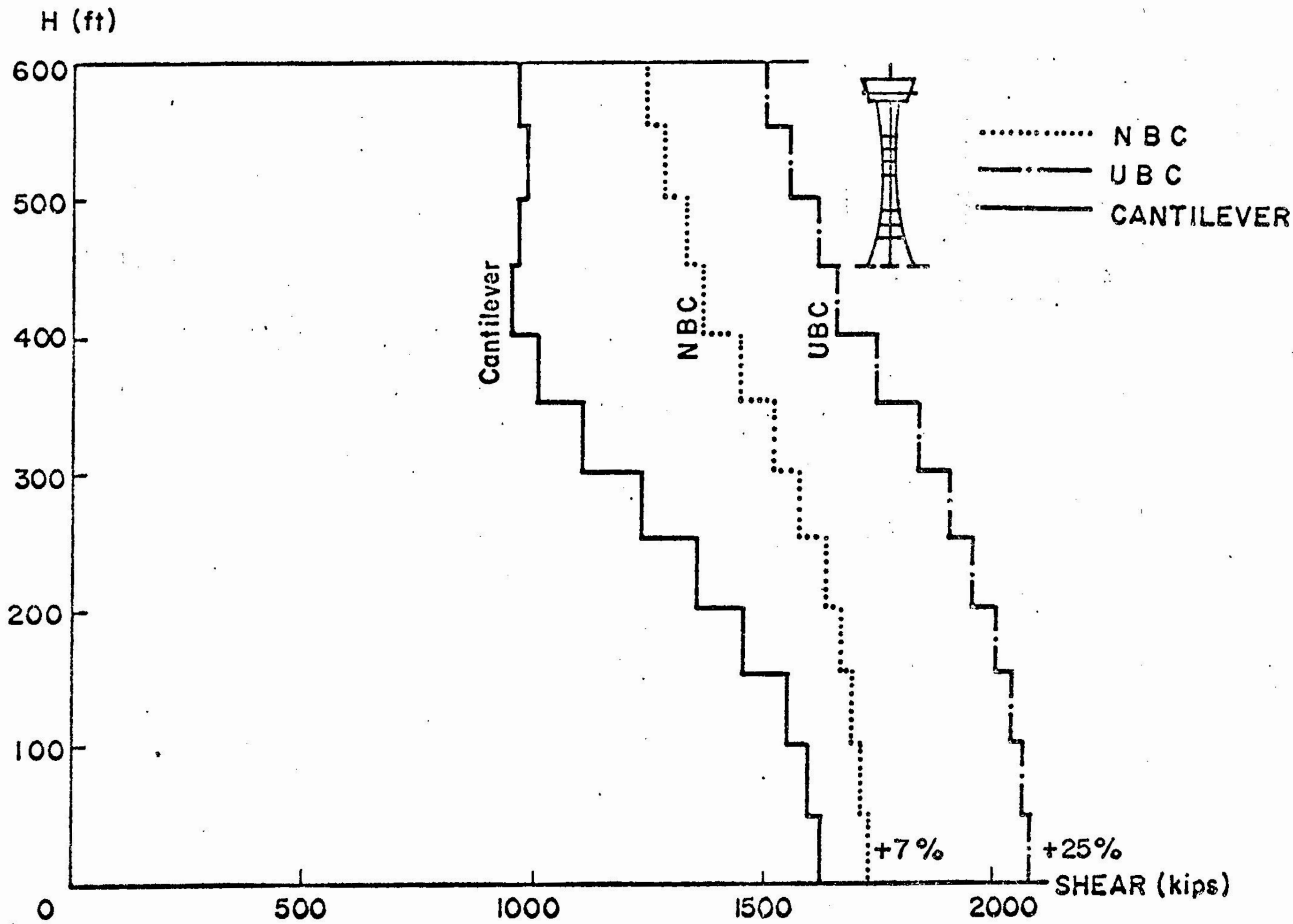


Fig.15 STRUCTURE No. 6

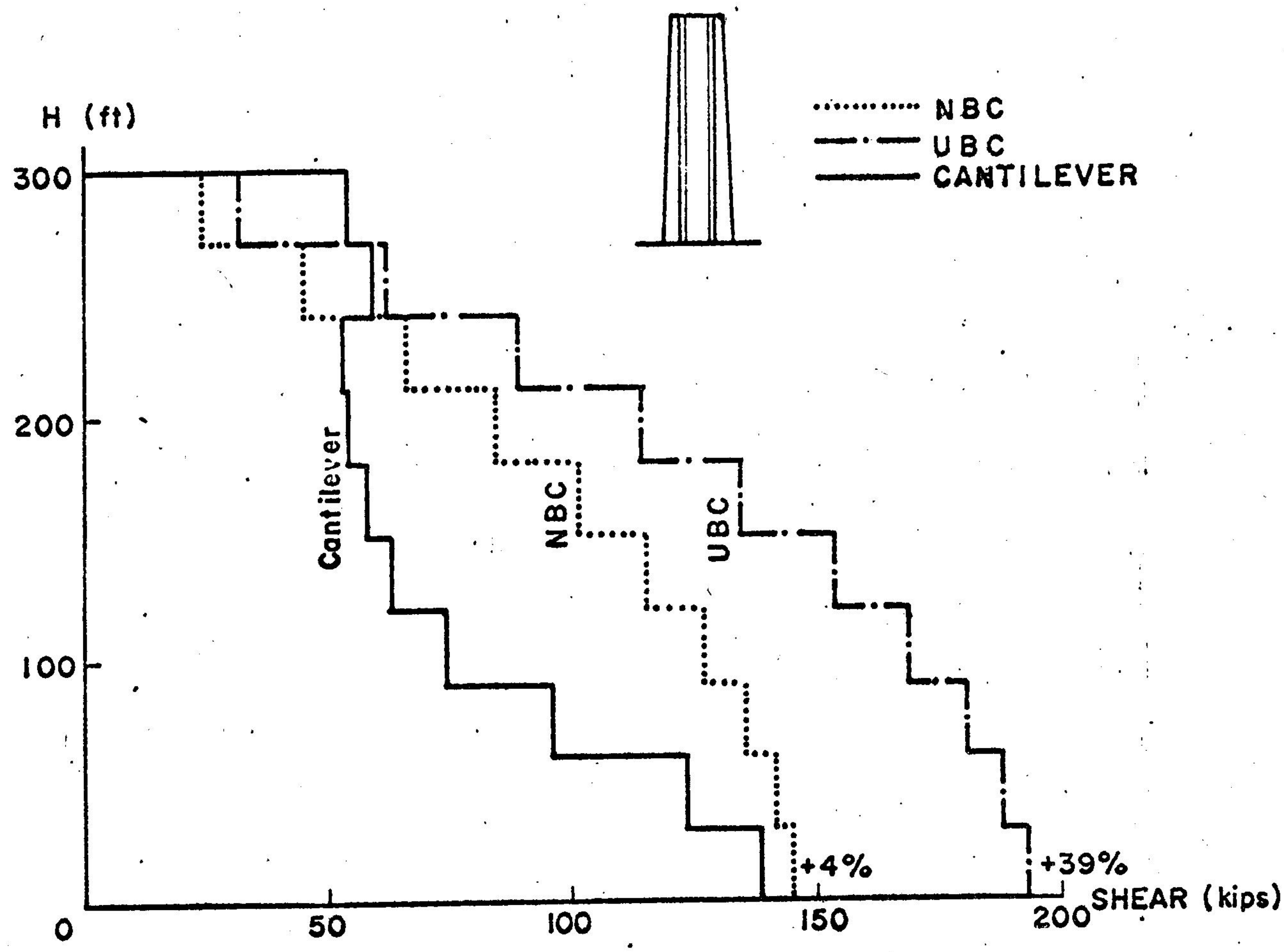


Fig.16 STRUCTURE No. 7